canonical units have been used, the value of the arc length or surface range between the two given points is given in earth radii

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Correlation of Flat-Plate Pressures Using the Rarefaction Parameter $M_\infty C_\infty^{1/2}/Re_{x_\infty}^{1/2}$

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Experimental flat-plate pressure data with apparent slip effects are presented A departure of $(\Delta p/p_\infty)/\bar{\chi}_\infty$ from the continuum flow constant value (γ = const, T_w/T_0 = const), which signifies the apparent onset of slip-flow effects, is shown to occur when $M_\infty C_\infty^{1/2}/Re_{x_\infty}^{1/2} \approx 0\ 1$ to $0\ 2$ The departure point appears to be independent of wall heating

RECENTLY, Talbot[1] showed the low-speed slip-flow parameter $M_\infty/\bar{R}e_{x_\infty}^{1/2}$ to be applicable to the hypersonic regime when $\bar{R}e_{x_\infty} = (\rho_\infty U_\infty X)/(\mu_\infty C_\infty)$, where $C_\infty = (\mu T_\infty)/(\mu_\infty T)$ He suggested that this parameter would be appropriate for correlation of experimental data in slip flow and pointed out that slip effects on surface pressure have been observed when $M_\infty C_\infty^{1/2}/Re_{x_\infty}^{1/2} \geq 0\ 10$ The purpose of this note is to present additional cold-wall, sharp, flat-plate pressure data in support of Talbot's observation and analysis

The experimental data presented in this note were obtained from recent tests in the Arnold Engineering Development Center 50-in hotshot tunnel[2] and the continuous-flow, low-density hypersonic tunnel (LDH)[3] at cold-wall conditions ($T_w/T_0 \simeq 0\ 1$) and from tests, reported by Aroesty,[4] in the University of California low-density wind tunnel (LDWT) at a hot-wall condition ($T_w/T_0 \simeq 1\ 0$) Most of the data of Nagamatsu et al [5] pertain completely to slip flow, and, therefore, deviation from continuum flow could not be observed; these data are not included

Strong viscous interaction theory[6] specifies that the induced pressure be a linear function of the viscous interaction parameter $\bar{\chi}_\infty$, where $\bar{\chi}_\infty = M_\infty^3/\bar{R}e_{x_\infty}^{1/2}$ The slope of the function in continuum flow is only dependent on the ratio of specific heats γ and the wall to total temperature ratio (T_w/T_0), e g, $m(\gamma, T_w/T_0) = (\Delta p/p_\infty)/\bar{\chi}_\infty$ The data are presented in terms of these parameters in Fig 1 When $\bar{\chi}_\infty$ is less than about 40, the hotshot experimental data support the linear relationship Two sets of data are plotted, one for a very sharp leading edge, $d = 0\ 001$ in, and one for a slightly more blunt leading edge, $d = 0\ 01$ in There is no difference (within the data scatter) in these data, which indicates that bluntness effects are completely overcome by the vis-

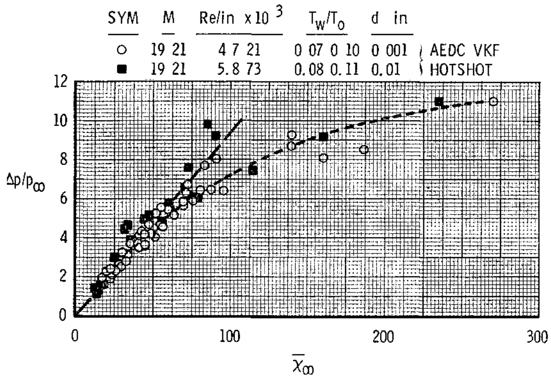

Fig 1 Sharp flat-plate pressure distribution

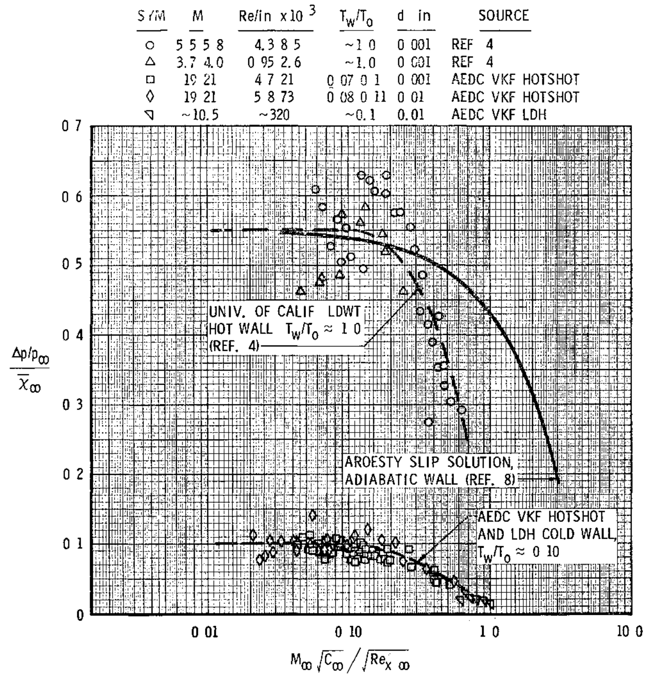

Fig 2 Effect of slip flow, pressure distribution

cous-induced pressure Of particular interest is the divergence from a linear relationship as $\bar{\chi}_\infty$ increases

This is brought out in Fig 2, which shows Aroesty's data[4] and the present experimental data plotted in terms of $(\Delta p/p_\infty)/\bar{\chi}_\infty$ and Talbot's slip parameter There is large scatter inherent in this type of figure, but it does not obscure nor alter the results For continuum flow, the ratio $(\Delta p/p_\infty)$ to $\bar{\chi}_\infty$ is independent of the slip parameter, whereas as slip effects are encountered it decreases with increasing slip parameter Both the hot-wall and the cold-wall data indicate the onset of slip effects at about $M_\infty C_\infty^{1/2}/Re_{x_\infty}^{1/2} \simeq 0\ 10$ to $0\ 20$ This is in excellent agreement with the observations of Talbot[1] and illustrates the dependence of slope on the slip parameter Vidal et al [7] observed a similar trend in their studies in the Cornell shock tunnel The initial departure from the continuum constant value appears to be independent of wall temperature, which is in agreement with Talbot Included is the Aroesty[8] solution of strong interactions with slip boundary conditions Although the theory is not adequate to predict quantitatively the slip effect, it does predict the correct trend, thus giving support to the use of the rarefaction parameter $M_\infty C_\infty^{1/2}/Re_{x_\infty}^{1/2}$

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Velocity Profiles from Compressible Wall Jets

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THE flow over afterbodies downstream from the fan-nozzle annulus of a fanjet engine is axially symmetric wall-jet flow As part of an experimental investigation to predict the drag on such afterbodies, the author has measured total pressure profiles in the flow over a 3-in -diam model, which consisted of a nosepiece, a convergent fan nozzle, a cylindrical afterbody, and a total pressure rake (Fig 1) The freestream air was supplied by an induction tunnel with a 20-in free jet

The flow from the convergent nozzle was compressible; the nozzle pressure ratio $(Pt/P)_f$ was varied from 2 0 to 4 4 The data are reported for $20 < X/h < 133$ and $2 < X/R_1 < 8$ The afterbody surfaces were smooth to 0 0001 in The freestream Mach number M_∞ was 0 and 0 8

Velocity Profiles for $M_\infty = 0$

The velocity U is defined as the fully expanded velocity from the indicated total pressure to the freestream static pressure A constant total temperature, usually equal to the nozzle total temperature, was assumed to exist throughout the mixing region The velocity profiles were nondimensionalized in the usual way: the radial distance from the afterbody surface r is divided by δ_2, which is the value of r at $(U_{max} + U_\infty)/2$; and U is divided by the maximum velocity for the profile, U_{max}

Figures 2 and 3 compare the present compressible data with Glauert's incompressible, two-dimensional profile of Ref 1 Over the range of geometries and pressure ratios tested, the experimental data closely follow Glauert's profile; it can be concluded that the effects of compressibility and axial symmetry are negligible

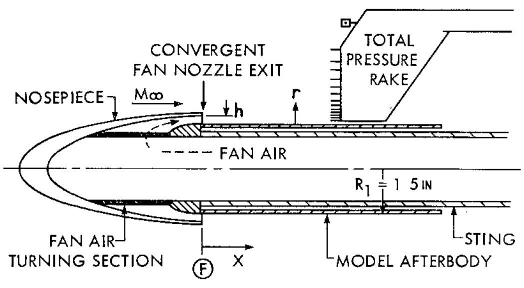

Fig 1 Model to simulate fan-nozzle annulus

Velocity Profiles for $M_\infty = 0\ 8$

The velocity profiles are given in terms of a velocity difference parameter $(U - U_\infty)/(U_{m\ x} - U_\infty)$ It is expected that, when $U_{max} \rightarrow U_\infty$, the profiles should be more strongly affected by skin friction; conversely, when $U_{max} \gg U_\infty$, the velocity difference between the fan exit velocity and U_∞ will be large, and the profiles should be strongly affected by mixing

Figure 4 is a case where $U_{m\ x} \gg U_\infty$ Since mixing predominates, the choice of $(U - U_\infty)/(U_{max} - U_\infty)$ for the

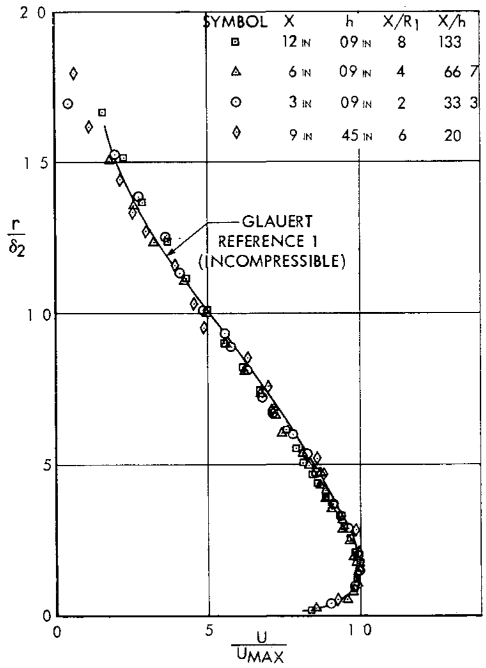

Fig 2 Axisymmetric compressible wall-jet velocity profiles; $M_\infty = 0$, $(Pt/P)_F = 2\ 0$, convergent nozzles

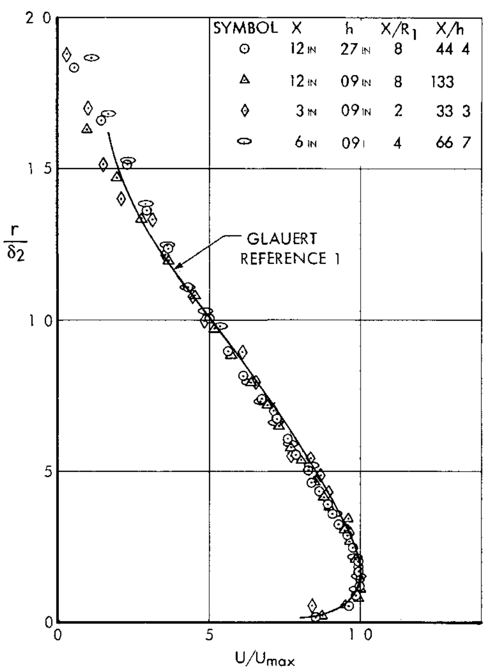

Fig 3 Axisymmetric compressible wall-jet velocity profiles; $M_\infty = 0$, $(Pt/P)_F = 4\ 4$, convergent nozzles

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